Exploring Global Stability in a Nonsmooth Dynamical Model of Hydraulic Cylinders

Bálint Magyar[†], **Gábor Stépán**[‡]

[†, ‡]Department of Applied Mechanics, Budapest University of Technology and Economics, Műegyetem rkp. 5, Budapest, H-1111, Hungary
[†]magyar@mm.bme.hu, [‡]stepan@mm.bme.hu

ABSTRACT

A highly simplified dynamic model of a hydraulic cylinder is investigated. It is assumed that the hydraulic fluid is incompressible and that the pump provides constant flow rates, which results in the possibility of velocity control. Two types of anomalies are taken into account: (a) the time delay due to the controller computations and the internal pressure dynamics and (b) the difference between maximum piston rod velocities up- and downwards. This results in a nonlinear system described by a piecewise linear discontinuous map. Nonlinear behaviour of the system is explored and the practically globally stable parameter domains are identified.

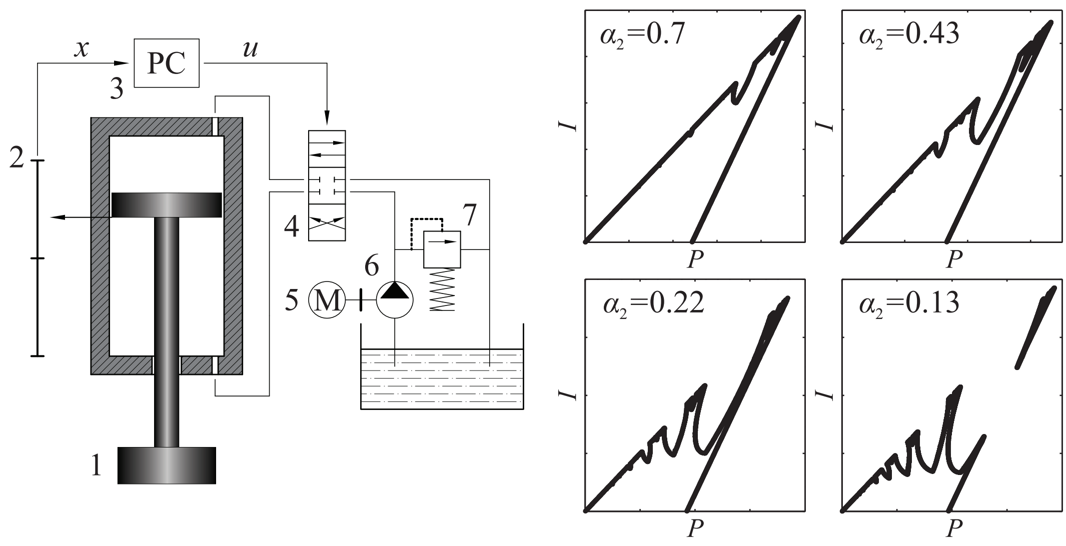

Figure 1: Left: the hydraulic positioning system. 1: hydraulic cylinder, 2: position transducer, 3: PC on which the PI controller is implemented, 4: proportional directional valve, 5: electro-motor, 6: gear pump, 7: pressure limiting valve. Right: stability charts for the PI controller with varying piston rod velocities (α_2).

Designing and tuning a PID controller of a hydraulic system is a highly challenging task mostly because the conventional ways are based on linear system theory. However, it is well-known that strong nonlinearities are present in these systems, such as pressure-flow rate relationship, dead zone of the control valves (see e.g. [1]), dry friction [2] or impact dynamics [3]. The discrete sampling time of the closed-loop control introduces additional complexity together with the response lag due to internal (mostly pressure) dynamics. Moreover, some of the above mentioned nonlinearities (e.g. piecewise linear behaviour) cannot be coped with using linearisation techniques.

The mathematical modelling of these systems often leads to equations with non-smooth or even discontinuous right-hand side. Fortunately, the progress in the theory of non-smooth dynamical systems (see e.g. [4] for an overview) provides a toolbox, albeit it is still far from being general. For example [5] gives a general theory on the existence of periodic and dense orbits for a bilinear one-dimensional map with a slight extension towards two-dimensional maps with delay and backlash. In [6] the authors study the border collision bifurcation in n-dimensional maps with two regions. Chaotic oscillations are also identified in these systems [7]. Based on numerical simulations, [8] describes an example on the effect of delay and backlash together.

Despite of its simplicity from the engineering point of view, the simplified piecewise smooth model of a hydraulic positioning system poses complex dynamical behaviour and interesting mathematical problems.

Acknowledgement. This work was supported in part by TÁMOP-4.2.2.B-10/1–2010-0009 and OTKA K 101714 grants.

References

[1] B. Magyar, C. Hős, G. Stépán, Influence of Control Valve Delay and Dead Zone on the Stability of a Simple Hydraulic Positioning System, *Mathematical Problems in Engineering*, Article ID: 349489, 2010.

[2] A.A. Manuel, A.G. Loukianov, E.N. Sanchez, Electrohydraulic actuator trajectory tracking, *Proceeding of the 2004 American Control Conference*, 2004.

[3] G. Licskó, A.R. Champneys, C. Hős, Dynamical analysis of a Hydraulic pressure relief valve, *Proceedings of the World Congress on Engineering*, 2009.

[4] M. di Bernardo, C.J. Budd, A.R. Champneys, P. Kowalczyk, Piecewise-smooth Dynamical Systems: Theory and Applications, Springer-Verlang London Limited, 2008.

[5] L. Kollár, G. Stépán, J. Turi: Dynamics of piecewise linear discontinuous maps, *International Journal of Bifurcation and Chaos*, Vol. 14, No. 7, 2341-2351, 2004.

[6] P.S. Dutta, B. Routroy, S. Banerjee, S.S. Alam, On the existence of low-period orbits in *n*-dimensional piecewise linear discontinuous maps, *Nonlinear Dyn*, 53: 369-380, 2008.

[7] P. Glendinning, P. Kowalczyk, Micro-chaotic dynamics due to digital sampling in hybrid systems of Filippov type, *Physica D*, 239(1-2), 58-71, 2010.

[8] L. Kollár, G.Stépán, J. Hogan, Sampling delay and backlash in balancing systems, *Periodica Politechnica*, Vol. 44, No. 1, 77-84, 2000.